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Barbara GŁADYSZ*
Dorota KUCHTA*

THE TWO-MACHINE OPEN SHOP PROBLEM WITH FUZZY PROCESSING TIMES

We consider a two-machine open shop scheduling problem in the case when the processing times of the jobs are not known exactly, but given only in the form of fuzzy numbers. In such a case the decision maker may be interested in the possible values of completing the whole processing and their possibility degrees. In the paper, we propose an answer to this question by defining the fuzzy optimal completion time.

1. Introduction

The paper is devoted to one of the scheduling problems, namely the two-machine open shop problem in case the processing times are not known exactly.

In many real-world scheduling problems processing times of jobs as well as possibly several other parameters are given in an imprecise form. Classical algorithms do not allow us to treat such decision problems. That is why it is necessary to search for models and algorithms helping to better evaluate cases containing imprecision and uncertainty. Here fuzzy numbers will be used to model imprecise processing times of jobs on machines. For each $t \in \Re$, the value of the corresponding membership function for t will show how it is possible that t will really be the respective processing time.

In the paper we will use the notion of fuzzy number – denoted by capital letters with a ~ (e.g., $\tilde{A}$), defined by its membership function μ_A – an upper-semicontinous function defined on the set of real numbers whose values belong to the interval [0, 1]. We will also use fuzzy operations on fuzzy numbers (i.e., addition and max, denoted by ~), defined on the basis of the well known Zadeh extension principle.

* Institute of Industrial Engineering and Management, Wrocław University of Technology, ul. Smoluchowskiego 25, 50-372 Wrocław, dorota.kuchta@pwr.wroc.pl, barbara.gladysz@pwr.wroc.pl

Contrary to the crisp case, in the fuzzy case we will not search for an optimal order of jobs. We will confine ourselves to determining the optimal fuzzy completion time, whose membership function will inform the decision maker which crisp optimal completion times may occur and which is their respective possibility degree.

2. The two-machine open shop problem in the crisp case

Let us consider a two-machine (denoted by M_1, M_2) system in which n jobs $J_1, J_2, ..., J_n$ should be processed. Each job J_i consists of two operations, one of which has to be processed on machine M_1 and the other one on machine M_2. The order in which the two operations should be executed is not prescribed, but the processing of one job on both machines is not allowed at the same moment.

For each job J_i the following information is given:

- a_i – the processing time on machine M_1,
- b_i – the processing time on machine M_2.

Let d denote the time when the last job leaves the system (all the jobs have been processed on both machines). The problem consists in determining such a processing order that d is minimal. This order will be called optimal order in the crisp case.

The optimal order can be found by Gonzalez and Sahni's algorithm from [5].

Theorem 1 [5]**.** The minimal total processing time d_{opt} can be determined by the following formula:

$$d_{\text{opt}} = \max\left(\sum_{i=1}^{n} a_i, \sum_{i=1}^{n} b_i, \max_{i=1,\dots n}(a_i + b_i)\right).$$

3. The two-machine open shop problem in the fuzzy case

Here the problem is stated as in section 2, the difference being such that the processing times are given in the form of fuzzy numbers ($\tilde{A}_i$, $\tilde{B}_i$).

We are interested in finding a fuzzy set $\tilde{D}_{\text{opt}}$ of optimal completion times, defined on the basis of the Zadeh extension principle:

$$\mu_{D_{\text{opt}}}(d) = \sup_{\substack{a_1,\dots,a_n,b_1,\dots,b_n \text{ such that } d=d_{\text{opt}} \\ \text{for crisp processing times} \\ a_1,\dots,a_n,b_1,\dots,b_n}} \min(\mu_{\tilde{A}_1}(a_1),\dots,\mu_{\tilde{A}_n}(a_n),\mu_{\tilde{B}_1}(b_1),\dots,\mu_{\tilde{B}_n}(b_n)) \quad (1)$$

The fuzzy set $\tilde{D}_{\text{opt}}$ will be called a fuzzy optimal completion time. It will provide the decision maker with information on what completion times he can expect – assuming that at the moment when exact values of the fuzzy processing times are known, he will be able to apply the optimal order corresponding to those crisp values.

Theorem 2. The fuzzy optimal completion time $\tilde{D}_{\text{opt}}$ can be determined by the following formula:

$$\tilde{D}_{\text{opt}} = \tilde{\max}\left(\tilde{\tilde{\sum_i}} \tilde{A}_i, \tilde{\tilde{\sum_i}} \tilde{B}_i, \tilde{\max_{i=1,\dots n}}\left(\tilde{A}_i + \tilde{B}_i\right)\right).$$

Proof follows immediately from Theorem 1 and the definition of fuzzy maximum and sum.

4. Numerical example

Let us consider three jobs J_1, J_2, J_3. For each job J_i its processing times ($\tilde{A}_i$, $\tilde{B}_i$) are triangular fuzzy numbers (Table 1).

Table 1

Fuzzy processing times for the example problem

Membership function \ Job	J_1	J_2	J_3
$\mu_{\tilde{A}_s}(a)$	(2,3,4)	(2,4,5)	(4,6,10)
$\mu_{\tilde{B}_s}(b)$	(1,2,3)	(2,5,6)	(8,10,11)

The fuzzy optimal completion time, determined according to Theorem 1, is shown in Figure 1.

Fig. 1. Fuzzy optimal completion time for the example

The membership function of $\tilde{D}_{\text{opt}}$ is as follows:

$$\mu_{D_{\text{opt}}}(d) = \begin{cases} 0 \text{ for } d \le 12 \text{ and for } d \ge 21 \\ \frac{1}{6}d - 2 \text{ for } 12 \le d \le 15 \\ 0{,}25d - 3{,}25 \text{ for } 15 \le d \le 17 \\ \frac{-1}{3}d + \frac{20}{3} \text{ for } 17 \le d \le 18.5 \\ \frac{-1}{5}d + \frac{21}{5} \text{ for } 18.5 \le d \le 21 \end{cases}$$

The optimal solution provides the decision maker with information on what completion times of all the jobs are possible and to which possibility degree. Assuming that it will be possible to behave rationally, i.e., apply the optimal order for the crisp values that will occur in reality, the decision maker knows which processing times he can expect and how much possible they are. This will help him to assess the risk of not finishing the processing of all the jobs in time.

5. Conclusions

In the paper, the two machine open shop problem with imprecise processing times has been considered. As scheduling usually concerns decisions which will be carried out only in the future, imprecision is a very common element of scheduling problems. The optimal solution gives the decision maker the knowledge about all possible total completion times of all the jobs which may occur and about their possibility degrees.

This knowledge will help the decision maker to assess the risk of not finishing the processing of all the jobs in time.

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Otwarty dwumaszynowy problem szeregowania z rozmytymi czasami obróbki

Rozważamy dwumaszynowy otwarty problem szeregowania, gdy czasy obróbki nie są znane dokładnie, lecz tylko dane w postaci liczb rozmytych. W takim przypadku pojęcie optymalnego uszeregowania i optymalnego czasu zakończenia są niejednoznaczne. W pracy nie podajemy definicji optymalnego rozmytego uszeregowania, lecz tylko definicję optymalnego rozmytego czasu zakończenia – jako zbioru wszystkich możliwych deterministycznych czasów zakończenia razem z ich stopniami możliwości. Pokazujemy, jak wyznaczyć tak zdefiniowany optymalny rozmyty czas zakończenia. Dostarczy on decydentowi informacji o tym, jakich czasów zakończenia może się spodziewać i w jakim stopniu są one możliwe. Rezultaty teoretyczne są zilustrowane przykładem liczbowym.